

2. Ekstremi funkcija više varijabli

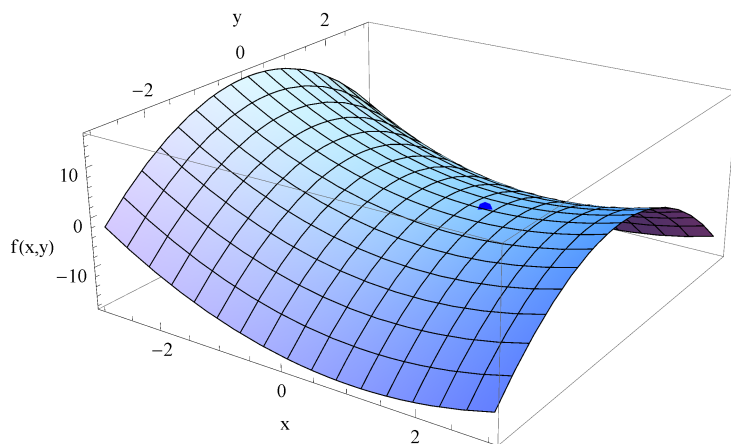
Plavo su označeni kandidati za ekstreme koji ne ispunjavaju dovoljne uvjete.
Crveno su označeni ekstremi.

IC 33. a)

```
Remove[f, xmin, xmax, ymin, ymax, Tstac, Textr];  
f[x_, y_] := (x - 1)^2 - 2 * y^2;  
xmin = -3;  
xmax = 3;  
ymin = -3;  
ymax = 3;  
Tstac = {{1, 0, f[1, 0]}};  
Textr = {{}};  
? f  
Show[  
  Plot3D[{f[x, y]}, {x, xmin, xmax}, {y, ymin, ymax}, AxesLabel -> {"x", "y", "f(x,y)"},  
  Graphics3D[{PointSize[Large], Blue, Point[Tstac]}],  
  Graphics3D[{PointSize[Large], Red, Point[Textr]}]  
]
```

Global`f

```
f[x_, y_] := (x - 1)^2 - 2 y^2
```



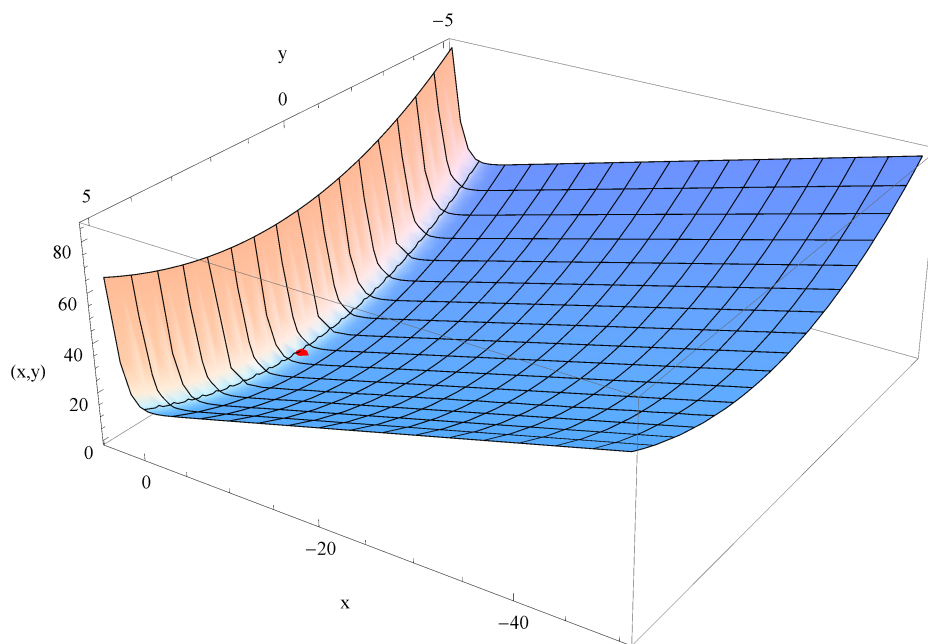
IC 33. b)

```

Remove[f, xmin, xmax, ymin, ymax, Tstac, Textr];
f[x_, y_] := Exp[x] - x + y^2 - 2 * y;
xmin = -50;
xmax = 4;
ymin = -5;
ymax = 5;
Tstac = {};
Textr = {{0, 1, f[0, 1]}};
? f
Show[
  Plot3D[{f[x, y]}, {x, xmin, xmax}, {y, ymin, ymax}, AxesLabel -> {"x", "y", "f(x,y)"},
  Graphics3D[{PointSize[Large], Blue, Point[Tstac]}],
  Graphics3D[{PointSize[Large], Red, Point[Textr]}]
]

```

Global`f

$$f[x_, y_] := e^x - x + y^2 - 2y$$


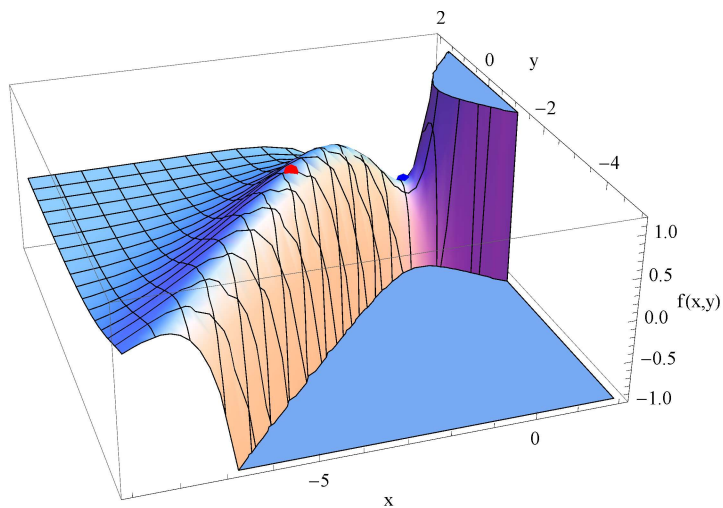
IC 33. c)

```

Remove[f, xmin, xmax, ymin, ymax, Tstac, Textr];
f[x_, y_] := Exp[x - y] * (x^2 - 2 * y^2);
xmin = -9;
xmax = 2;
ymin = -5;
ymax = 2;
Tstac = {{0, 0, f[0, 0]}};
Textr = {{-4, -2, f[-4, -2]}};
? f
Show[
  Plot3D[{f[x, y]}, {x, xmin, xmax}, {y, ymin, ymax},
    AxesLabel -> {"x", "y", "f(x,y)"}, PlotRange -> {-1., 1.2}],
  Graphics3D[{PointSize[Large], Blue, Point[Tstac]}],
  Graphics3D[{PointSize[Large], Red, Point[Textr]}]
]

```

Global`f

$$f[x_, y_] := e^{x-y} (x^2 - 2y^2)$$


IC 33. e)

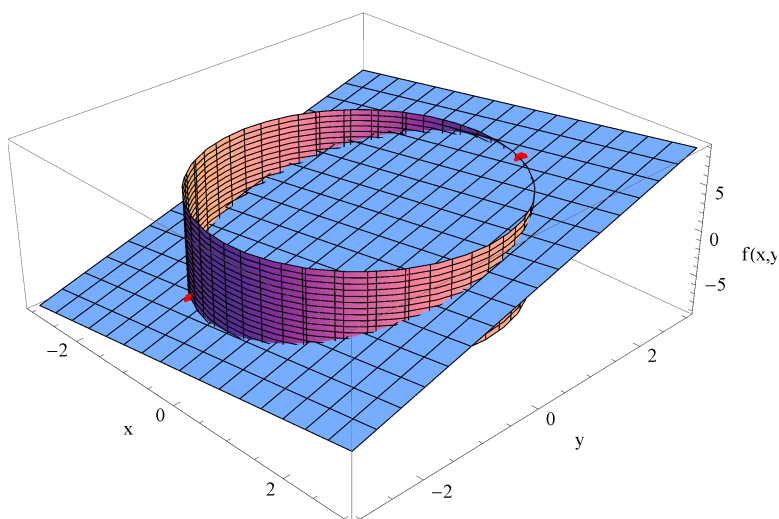
```

Remove[f, xmin, xmax, ymin, ymax, Tstac, Textr];
f[x_, y_] := x + 2 y;
xmin = -3;
xmax = 3;
ymin = -3;
ymax = 3;
Tstac = {{}};
Textr = {{-1, -2, f[-1, -2]}, {1, 2, f[1, 2]}};
?f
Show[
  Plot3D[{f[x, y]}, {x, xmin, xmax}, {y, ymin, ymax}, AxesLabel -> {"x", "y", "f(x,y)"},
  Graphics3D[{PointSize[Large], Blue, Point[Tstac]}],
  Graphics3D[{PointSize[Large], Red, Point[Textr]}],
  ContourPlot3D[x * x + y * y - 5 == 0, {x, xmin, xmax}, {y, ymin, ymax}, {z, -5, 5}]
]

```

Global`f

f[x_, y_] := x + 2 y



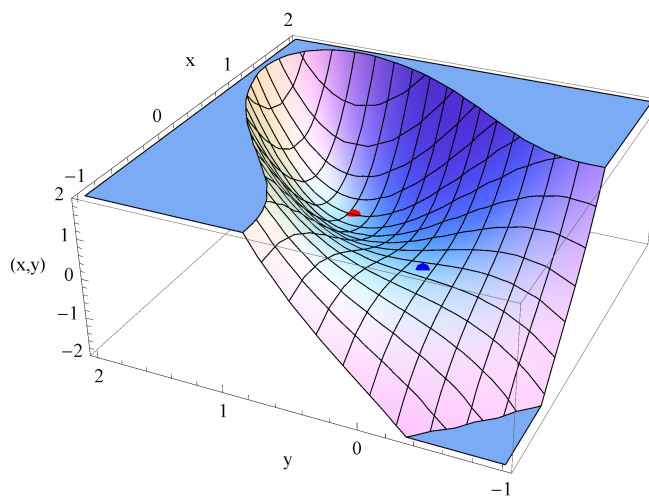
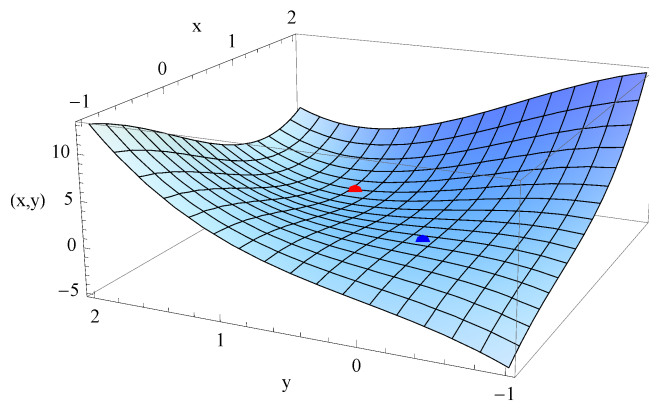
PS 33. h)

```

Remove[f, xmin, xmax, ymin, ymax, Tstac, Textr];
f[x_, y_] := x^3 + y^3 - 3 * x * y; ? f
xmin = -1; xmax = 2; ymin = -1; ymax = 2;
Tstac = {{0, 0, f[0, 0]}};
Textr = {{1, 1, f[1, 1]}};
Show[
  Plot3D[{f[x, y]}, {x, xmin, xmax}, {y, ymin, ymax}, AxesLabel → {"x", "y", "f(x,y)"},
  Graphics3D[{PointSize[Large], Blue, Point[Tstac]}],
  Graphics3D[{PointSize[Large], Red, Point[Textr]}] ]
Show[
  Plot3D[{f[x, y]}, {x, xmin, xmax}, {y, ymin, ymax},
  AxesLabel → {"x", "y", "f(x,y)"}, PlotRange → {-2., 2}],
  Graphics3D[{PointSize[Large], Blue, Point[Tstac]}],
  Graphics3D[{PointSize[Large], Red, Point[Textr]}] ]

```

Global`f

$$f[x_, y_] := x^3 + y^3 - 3xy$$


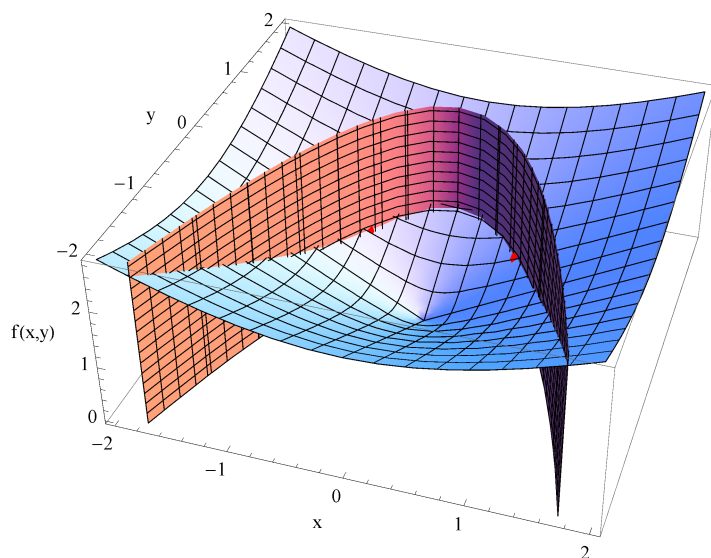
IC 34.

```

Remove[f, xmin, xmax, ymin, ymax, Tstac, Textr];
f[x_, y_] := Sqrt[x^2 + y^2];
xmin = -2;
xmax = 2;
ymin = -2;
ymax = 2;
Tstac = {{}};
Textr = {{1 / Sqrt[2], 0.5, f[1 / Sqrt[2], 0.5]}, {-1 / Sqrt[2], 0.5, f[1 / Sqrt[2], 0.5]}};
?f
Show[
  Plot3D[{f[x, y]}, {x, xmin, xmax}, {y, ymin, ymax}, AxesLabel -> {"x", "y", "f(x,y)"},
  Graphics3D[{PointSize[Large], Blue, Point[Tstac]}],
  Graphics3D[{PointSize[Large], Red, Point[Textr]}],
  ContourPlot3D[x * x + y - 1 == 0, {x, xmin, xmax}, {y, ymin, ymax}, {z, 0, 3}]
]

```

Global`f

$$f[x_, y_] := \sqrt{x^2 + y^2}$$


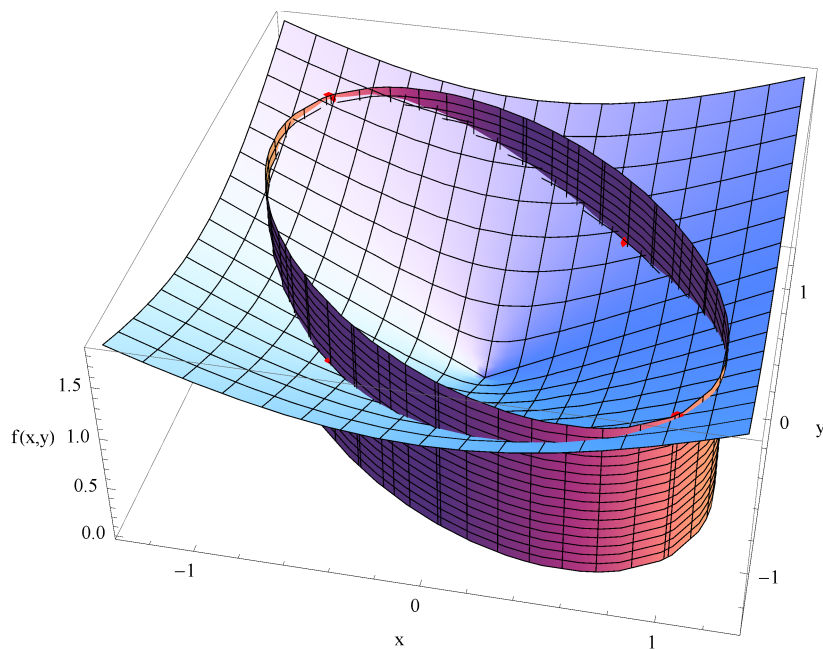
IC 35.

```

Remove[f, xmin, xmax, ymin, ymax, Tstac, Textr];
f[x_, y_] := Sqrt[x^2 + y^2];
xmin = -1.3;
xmax = 1.3;
ymin = -1.3;
ymax = 1.3;
Tstac = {{}};
Textr = {{Sqrt[3] / 3, Sqrt[3] / 3, f[Sqrt[3] / 3, Sqrt[3] / 3]},
        {-Sqrt[3] / 3, -Sqrt[3] / 3, f[-Sqrt[3] / 3, -Sqrt[3] / 3]},
        {1, -1, f[1, -1]}, {-1, 1, f[-1, 1]}};
? f
Show[
  Plot3D[{f[x, y]}, {x, xmin, xmax}, {y, ymin, ymax}, AxesLabel -> {"x", "y", "f(x,y)"},
  Graphics3D[{PointSize[Large], Blue, Point[Tstac]}],
  Graphics3D[{PointSize[Large], Red, Point[Textr]}],
  ContourPlot3D[x^2 + x*y + y^2 - 1 == 0, {x, xmin, xmax}, {y, ymin, ymax}, {z, 0, 1.45}]
]

```

Global`f

$$f[x_, y_] := \sqrt{x^2 + y^2}$$


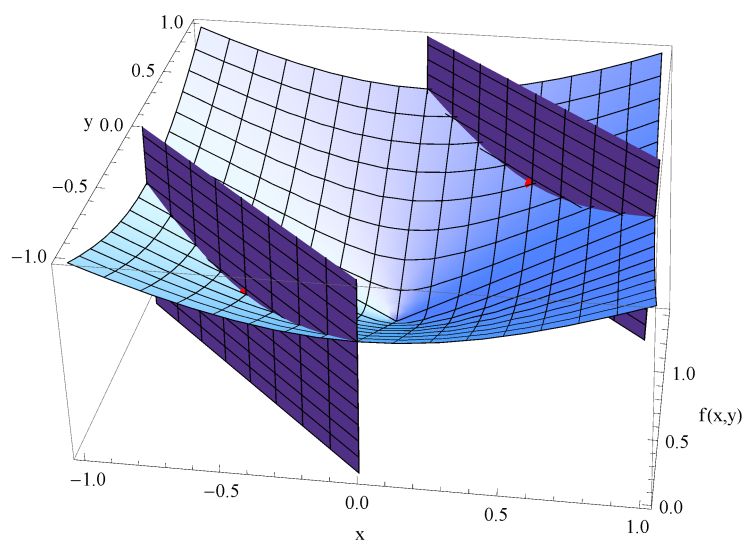
IC 36.

```

Remove[f, xmin, xmax, ymin, ymax, Tstac, Textr];
f[x_, y_] := Sqrt[x^2 + y^2];
xmin = -1;
xmax = 1;
ymin = -1;
ymax = 1;
Tstac = {{}};
Textr = {{0.5, 0.5, f[0.5, 0.5]}, {-0.5, -0.5, f[-0.5, -0.5]}};
?f
Show[
  Plot3D[{f[x, y]}, {x, xmin, xmax}, {y, ymin, ymax}, AxesLabel -> {"x", "y", "f(x,y)"},
  Graphics3D[{PointSize[Large], Blue, Point[Tstac]}],
  Graphics3D[{PointSize[Large], Red, Point[Textr]}],
  ContourPlot3D[x^2 + 2 * x * y + y^2 - 1 == 0, {x, xmin, xmax}, {y, ymin, ymax}, {z, 0, 2}]
]

```

Global`f

$$f[x_, y_] := \sqrt{x^2 + y^2}$$


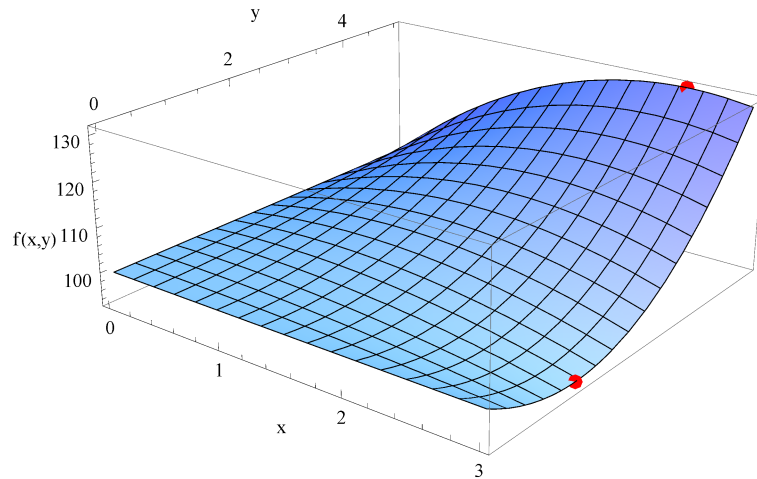
IC 39.

```

Remove[f, xmin, xmax, ymin, ymax, Tstac, Textr];
f[x_, y_] := x*y^2 - x^2*y + 100;
xmin = 0;
xmax = 3;
ymin = 0;
ymax = 5;
Tstac = {{}};
Textr = {{3, 3/2, f[3, 3/2]}, {5/2, 5, f[5/2, 5]}};
?f
Show[
  Plot3D[{f[x, y]}, {x, xmin, xmax}, {y, ymin, ymax}, AxesLabel -> {"x", "y", "f(x,y)"},
  Graphics3D[{PointSize[Large], Blue, Point[Tstac]}],
  Graphics3D[{PointSize[Large], Red, Point[Textr]}]
]

```

Global`f

$$f[x_, y_] := x y^2 - x^2 y + 100$$


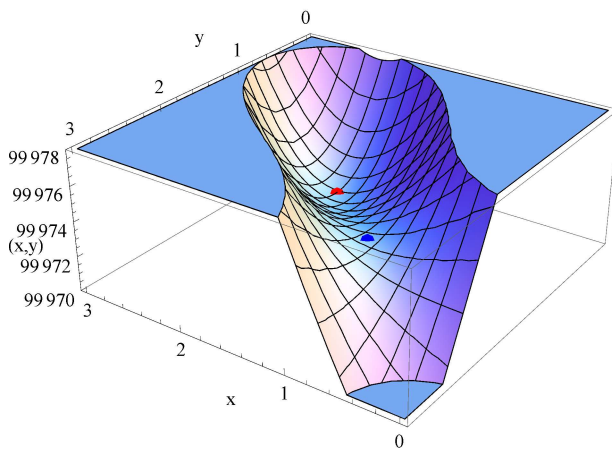
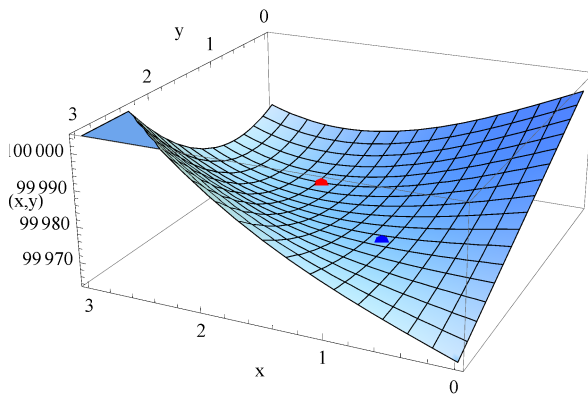
K0910.1 A

```

Remove[f, xmin, xmax, ymin, ymax, Tstac, Textr];
f[x_, y_] := 10^5 + (x^3 + 3 * x * y^2) - (15 * x + 12 * y); ? f
xmin = 0; xmax = 3; ymin = 0; ymax = 3;
Tstac = {{1, 2, f[1, 2]}};
Textr = {{2, 1, f[2, 1]}};
Show[
  Plot3D[{f[x, y]}, {x, xmin, xmax}, {y, ymin, ymax}, AxesLabel -> {"x", "y", "f(x,y)"},
  Graphics3D[{PointSize[Large], Blue, Point[Tstac]}],
  Graphics3D[{PointSize[Large], Red, Point[Textr]}]
]
Show[
  Plot3D[{f[x, y]}, {x, xmin, xmax}, {y, ymin, ymax},
  AxesLabel -> {"x", "y", "f(x,y)"}, PlotRange -> {99970, 99978}],
  Graphics3D[{PointSize[Large], Blue, Point[Tstac]}],
  Graphics3D[{PointSize[Large], Red, Point[Textr]}]
]

```

Global`f

$$f[x_, y_] := 10^5 + (x^3 + 3 x y^2) - (15 x + 12 y)$$


K0910.1 B

```

Remove[f, xmin, xmax, ymin, ymax, Tstac, Textr];
f[x_, y_] := 5 - 4 * x - 3 * y;
xmin = -1.2;
xmax = 1.2;
ymin = -1.2;
ymax = 1.2;
Tstac = {};
Textr = {{4 / 5, 3 / 5, f[4 / 5, 3 / 5]}, {-4 / 5, -3 / 5, f[-4 / 5, -3 / 5]}};
?f
Show[
  Plot3D[{f[x, y]}, {x, xmin, xmax}, {y, ymin, ymax}, AxesLabel -> {"x", "y", "f(x,y)"},
  Graphics3D[{PointSize[Large], Blue, Point[Tstac]}],
  Graphics3D[{PointSize[Large], Red, Point[Textr]}],
  ContourPlot3D[x * x + y * y - 1 == 0, {x, xmin, xmax}, {y, ymin, ymax}, {z, 0, 10}]
]

```

Global`f

f[x_, y_] := 5 - 4 x - 3 y

